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THE GEOMETRY OF PRINCIPAL AND AGENT: YET ANOTHER USE FOR THE EDGEWORTH BOX

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I

INTRODUCTION

In recent years much theoretical work has been devoted to the problem of principal and agent (e.g. Harris and Raviv, 1978; Holmström 1979; and Shavell, 1979). The result has been both great theoretical insight and the opportunity to apply some of the new ideas to the analysis of such problems as the nature of contracts of employment, the structure of hierarchical organisations, the working of insurance markets, the "division of ownership from control," the monitoring of government regulations, and so forth. The purpose of this paper is to use some familiar diagrammatic tools to illustrate some of the main results of principal-agent theory.

II

THE PRINCIPAL-AGENT PROBLEM

A principal-agent relation exists when one party (the agent) agrees to act on behalf of another party (the principal). Although principal-agent relationships are widespread, they face obvious difficulties. How is the principal to know whether the agent is doing what he contracted to do? Clearly this is no problem if the principal is sure that the agent has identical objectives. There is, in other words, no conflict of interest. The agent serves the principal whenever he serves himself. Neither is there a problem if the principal has perfect information about the behaviour of the agent, since the punishment for departure from the pursuit of the principal's interests would be easy to write into a contract. In general, however, neither of these conditions is fulfilled. The agent is less than perfectly "observable" and his interests may not coincide with those of his principal.

The problem then exists of devising a contract which provides incentives for the agent to work in ways which benefit the principal. When the

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behaviour of the agent is deterministically related to the final outcome, the "unobservability" of the agent's behaviour is obviously of no consequence so long as the outcome can be easily observed, and we are back to the case of a perfectly normal contractual relation. When, however, the outcome is stochastic, and the agent's behaviour affects the probability distribution of outcomes, knowledge of the outcome alone will not be sufficient to deduce the agent's behaviour. The provision of incentives will then require that the agent's remuneration vary depending upon what outcome occurs. The agent, in other words, will bear some of the risk associated with variations in the final outcome. Clearly, one contract which would give to the agent an incentive to ensure "good" outcomes would be for the agent to bear the entire risk and simply pay a fixed sum to the principal. Except in the special case of risk neutrality on the part of the agent, however, this will imply an inefficient distribution of risk bearing between principal and agent. Thus a trade-off exists between gains from the provision of extra incentives and losses from inefficient risk sharing. It is this problem which is illustrated diagrammatically in the sections which follow.

It is usual in principal-agent theory to consider the "outcome" (π) of a contract as depending upon a random "state of the world" (θ) and the agent's effort (ω). Efficient contracts are then specified according to what information is assumed to be available. We shall be concerned with the basic case in which neither the "state of the world" nor the agent's "effort" is observable. The outcome alone is known to both parties. A "contract" will simply consist therefore, of a specified share of the outcome assigned to the parties for the various outcomes which are possible. The simplification which permits the use of diagrams is the assumption that only two outcomes are possible (π_1 and π_2). An Edgeworth box diagram can then be used to illustrate the following propositions:-

(1) If the agent is risk averse an efficient contract will nevertheless require the agent to bear some risk.

(2) If the agent is risk averse an efficient contract can never involve the agent shouldering the entire risk.

(3) If the agent is risk neutral an efficient contract will involve the agent bearing the entire risk. There is then no potential gain to observing the agent's behaviour.

(4) Where effort efficiency on the part of the agent is very great the Pareto efficient contract will be almost "first best" from a risk sharing point of view.

(5) Where both principal and agent are risk averse the Pareto efficient contract will never be first-best. Some of the advantages of risk sharing will be sacrificed in the interest of eliciting greater effort from the agent.

(6) Where the principal is slightly risk preferring and the agent is risk averse, even perfectly efficient effort of the part of the agent will not permit a first best contract.

(7) Where the agent is slightly risk preferring and the principal is risk averse, the principal will bear some risk but the Pareto efficient contract will not be first best.

Propositions (1)–(5) have been formally proved by Shavell (1979) and by Holmström (1979). The diagrams developed here apply to a more restricted class of utility function but the analysis is related closely to Holmström's approach.

III

Risk Sharing

Consider a simple two person, two outcome model. Both persons are expected utility maximisers. The probabilities associated with the two outcomes are known. Let π_{1A} be the money payoff to person A in the event of outcome 1. Let q_1 be the probability of outcome 1. We can then write

$$E[U_A(\pi_A)] = q_1 U_A(\pi_{1A}) + (1 - q_1) U_A(\pi_{2A}),$$

where U_A is a von Neumann–Morgenstern utility function and E is the expectations operator.

Setting $dE[U_A(\pi_A)] = 0$ we derive an expression for the slope of person A's constant expected utility indifference curve

$$\frac{\partial \pi_{2A}}{\partial \pi_{1A}} = \frac{-q_1}{1 - q_1} \frac{U'_A(\pi_{1A})}{U'_A(\pi_{2A})}.$$

In the case of another person P we would similarly have

$$\frac{\partial \pi_{2p}}{\partial \pi_{1p}} = \frac{-q_1}{1 - q_1} \frac{U'_p(\pi_{1p})}{U'_p(\pi_{2p})}.$$

The combined payoff to persons A and P we call an "outcome." Where there are just two outcomes we can proceed to construct an Edgeworth box entirely analogous to the apparatus used in elementary textbooks to illustrate the gains from exchange in a non-stochastic world.

In Figure 1, the horizontal dimension of the box is outcome 1. The vertical distance is outcome 2. Any point in the box represents a "contract" i.e. a division of the total payoff between A and P for each outcome. A's indifference curves are designated $\bar{U}_A$, $\bar{\bar{U}}_A$. Convexity of A's indifference curves implies that A is risk averse. P's indifference curves are drawn as straight lines $\bar{U}_p$, $\bar{\bar{U}}_p$. This is the special case of risk neutrality. Clearly, if the two individuals can trade costlessly they will achieve an efficient contract along CC'. This use of the Edgeworth box diagram is standard in microeconomics texts (e.g. Gravelle and Rees, 1981, pp. 577–581).

Because person P is risk neutral, a typical P indifference curve will have a slope of $-q_1/1 - q_1$ along its entire length. It is important for the future analysis to note that where $\pi_{1A} = \pi_{2A}$ and hence where A's payoff is independent of the outcome, person A's indifference curves will also have a slope of $-q_1/1 - q_1$. In Figure 1 therefore, along the 45° line from A's origin, A's strictly convex indifference curves will be tangential to P's straight ones.

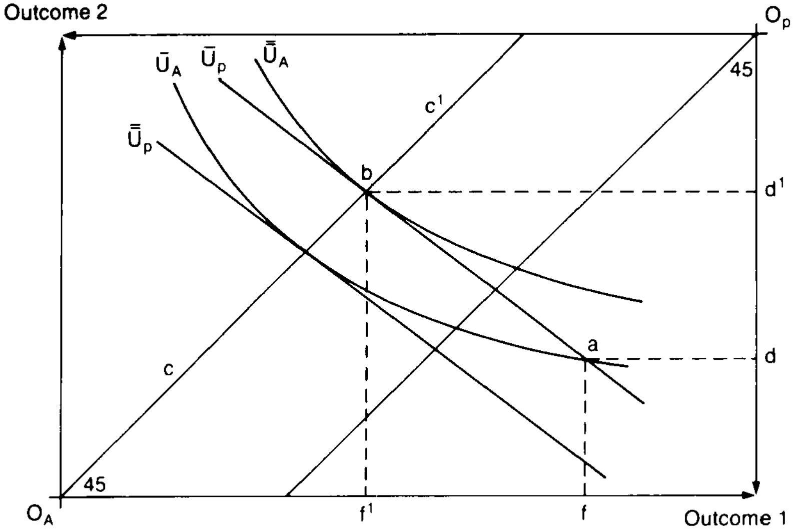

Figure 1. *A* is risk averse. *P* is risk neutral. Efficient risk-sharing occurs along *CC'*.

Diagrammatically this verifies what is intuitively obvious, that if one person is risk averse and the other risk neutral, the latter should bear all the risk and insure the former against any variation in the outcome. Suppose that *A* and *P* start at point *a* in Figure 1. Clearly, a move from *a* to *b* would represent a Pareto improvement. *P* sacrifices *d'd* claims contingent on outcome 2 in exchange for *ff'* claims contingent on outcome 1. This ratio of exchange is precisely equal to the ratio of outcome probabilities. *P* is therefore providing *A* with "fair" insurance. "Fair" insurance will always be accepted by risk averse *A* and will not hurt risk neutral *P*.

IV

EFFORT AND INCENTIVES

We now introduce the agency problem by assuming that one of the parties (person A) is acting on behalf of the other (person *P*). Person *A* we continue to assume is risk averse, while person *P* is risk neutral. Let us further assume that the agent *A* can, be exerting effort *e*, increase the probability that outcome 1 occurs. Effort level e_1 will result in a probability of outcome 1 of $q_1^{(1)}$, effort level e_2 a probability of $q_1^{(2)}$, e_3 of $q_1^{(3)}$ and so on, where $q_1^{(1)} < q_1^{(2)} < q_1^{(3)}$.

This change in the probability distribution of outcomes induced by *A*'s effort will affect the slopes of all the indifference curves of both *A* and *P*. In particular, along a 45° line through person *A*'s origin, all indifference curves relating to person *A* will have a slope of $-q_1^{(n)}/1 - q_1^{(n)}$ where *n* is the degree of effort. The same applies to *P*'s indifference curves along *P*'s 45° line.

Figure 2 illustrates the effect of effort on A's indifference map. At effort level e_0, person A achieves utility index $\bar{U}_A$ all along the curve labelled $\bar{U}_A^{e'_0}$. Effort level $e_1 > e_0$ changes the ratio of probabilities and person A now achieves utility index $\bar{U}_A$ along the curve labelled $\bar{U}_A^{e'_1}$. Along the 45° line $\bar{U}_A^{e'_1}$ is steeper than $\bar{U}_A^{e'_0}$. The "prime" on the letter e indicates that it is person A who is exerting the effort. If someone other than A took the action necessary to change the ratio of outcome probabilities, person A's new indifference curve $\bar{U}_A^{e_1}$ would cut the old one $\bar{U}_A^{e'_0}$ at θ along the 45° line. The distance $\theta\varepsilon$ in the diagram is a measure of the cost of additional effort to person A.

Consider now the point of intersection between $\bar{U}_A^{e'_0}$ and $\bar{U}_A^{e'_1}$. At this point, person A has a portfolio of outcome contingent claims such that he is just indifferent between effort e'_0 and effort e'_1. Any point to the right of r between curves $\bar{U}_A^{e'_1}$ and $\bar{U}_A^{e'_0}$ will ensure that A strictly prefers effort e'_1 over effort e'_0. Indeed we can imagine many points of intersection between indifference curves with superscripts e'_1 and e'_0 representing different utility indices. r' for example, is the point of intersection of $\bar{\bar{U}}_A^{e'_1}$ and $\bar{\bar{U}}_A^{e'_0}$ where $\bar{\bar{U}}_A > \bar{U}_A$. By joining points such as r and r' we derive a line labelled $q_1^{(1)}$. Contracts along $q_1^{(1)}$ will just leave person A at the margin of doubt as to whether to exert effort e_1 or to stay at effort level e_0. The circumstances in which $q_1^{(1)}$ can be drawn as a straight line are discussed in Appendix 1.

A similar argument can be used to derive another curve, the locus of points of intersection of indifference curves representing the same utility index but with superscripts e'_1 and e'_2. Along $q_1^{(2)}$ person A is just indifferent between effort level e'_1 and e'_2. The curve $q_1^{(2)}$ will lie to the right of $q_1^{(1)}$ under weak assumptions (see Appendix). The intuition behind this construction is not difficult. In order to get A to exert effort in the interest of increasing the

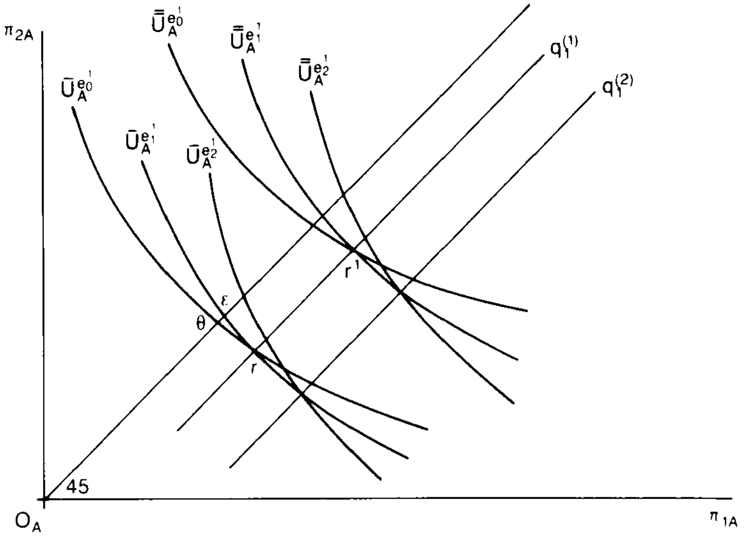

Figure 2. Along $q_1^{(1)}$ person A is indifferent between effort e_0 and effort e_1. Along $q_1^{(2)}$ person A is indifferent between effort e_1 and effort e_2.

chances of outcome 1, we have to give *A* a reasonable stake in this outcome. Along the 45° line, *A* does not care whether outcome 1 or outcome 2 occurs. Only by giving *A* a greater portfolio of outcome 1 claims relative to outcome 2 claims can we induce voluntary (non-monitored) effort. With increasing marginal disutility of effort and a diminishing influence of effort on the ratio of outcome probabilities, each successive increment of effort can be induced only by moving larger successive distances from *A*'s 45° line.

The effect of *A*'s effort on the indifference curves of the principal *P* is illustrated in Figure 3. We assume that principal and agent start at θ'. At θ' the agent has the same money payoff irrespective of which outcome occurs. The principal's income varies and is higher in the case of outcome 1 compared with outcome 2. As the probability of outcome 1 increases, *P*'s indifference curves will become steeper. At any *given* point on *P*'s 45° line however, expected utility will be constant irrespective of the probability distribution of outcomes. Suppose that *P*'s original indifference curve $\bar{U}_p^{e_0}$ through θ' cuts his 45° line at ϕ. We may then imagine a whole family of indifference curves representing the same expected utility index but applying to different ratios of state probabilities and radiating outward from ϕ. These indifference curves are labelled $\bar{U}_p^{e_0}$, $\bar{U}_p^{e_1}$, $\bar{U}_p^{e_2}$, $\bar{U}_p^{e_3}$ in Figure 3.

It is now possible to superimpose the lines $q_1^{(1)}$, $q_1^{(2)}$, $q_1^{(3)}$ etc. from Figure 2 on Figure 3. Contracts along $q_1^{(1)}$ will just persuade the agent to put in effort e_1. The intersection point between $q_1^{(1)}$ and $\bar{U}_p^{e_1}$ will therefore define a contract giving a utility index $\bar{U}_p$ to the principal. Similarly, the point of intersection between $q_1^{(2)}$ and $\bar{U}_p^{e_2}$ will also define a contract yielding utility index $\bar{U}_p$ to the principal. Joining up these points of intersection yields the locus $\theta\bar{P}$. Along $\theta\bar{P}$ the principal's utility is constant.

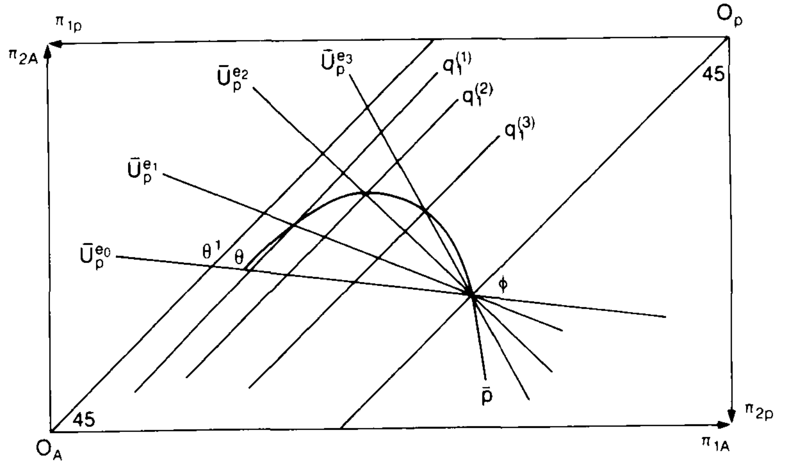

Figure 3. Contracts along $\theta\bar{P}$ yield the same utility index to *P* while inducing different levels of effort from *A* and hence different outcome probabilities.

V

Efficient Contracts

To identify an efficient contract we simply maximise the agent's utility while maintaining the utility of the principal constant. Figure 4 illustrates an efficient contract at a point such as ω where $\bar{U}_A^{e_n'}$ is tangential to $\theta\bar{P}$ at a point of intersection with $q_1^{(n)}$. That a point of tangency such as occurs at ω represents an efficient contract can be confirmed by supposing that an indifference curve such as $\bar{U}_A^{e_m'}$ cuts the locus $\theta\bar{P}$ from above (say at α). If this were the case there must exist a contract involving a higher effort level on the part of A which would be Pareto preferred. In the figure any contract along the line $q_1^{(n)}$ between β and ω is Pareto preferred to a contract at α.

Assuming that $\bar{U}_A$ represents the agent's utility index at θ', the locus of Pareto efficient contractual arrangements which are Pareto preferred to θ' will be given by the line between β and ω in Figure 4. This figure clearly indicates the distinction drawn in principal-agent theory between Pareto efficient contracts and first best solutions. "First best" solutions lie along the contract line CC' (in this case A's 45° line) and involve the efficient sharing of risk. They are, however, entirely unattainable in the postulated circumstances in which the agent's behaviour cannot be observed. The locus $\beta\omega$ is by contrast the best we can do in the circumstances, although obviously not as good as would be possible if only we could assume away the problem

We are now in a position to explore diagrammatically some of the results of principal-agent theory mentioned earlier:

(1) If there are Pareto improvements available from the employment of an agent, an efficient contract will lie somewhere along $\theta\bar{P}$. At θ' both parties

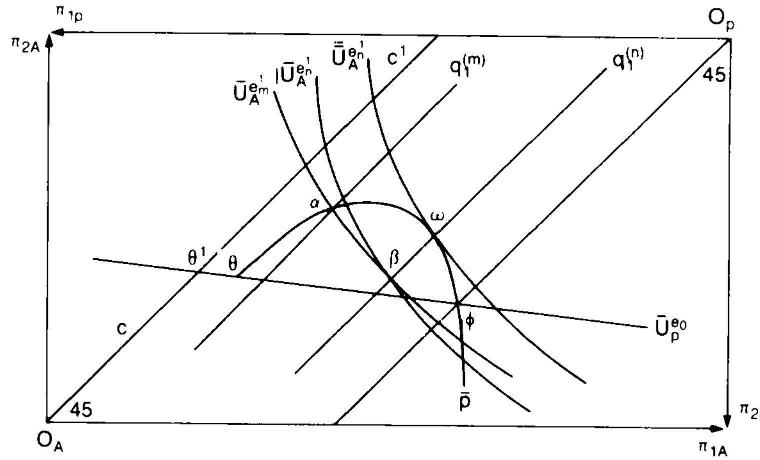

Figure 4. A Pareto efficient contract is given at ω. If an 'A' indifference curve applying to effort level e_m' cuts the locus $\theta\bar{P}$ from above at a point such as α, a higher level of effort and hence a contract between β and ω will be Pareto preferred.

are at their original levels of utility. Thus, if effort is costly to A, the efficient contract will always involve the agent in some risk bearing.

(2) The agent will never bear the entire risk. If the agent bore the entire risk this would imply that an efficient contract was possible at ϕ. Figure 5 indicates, however, that at ϕ the principal's indifference curve $\bar{U}_p^{e_v}$ is tangential to the locus $\theta\bar{P}$. Given that a contract at ϕ induces effort level e'_v on the part of the agent we draw $\bar{U}_A^{e'_v}$ through ϕ. This curve will always *cut* $\theta\bar{P}$ from below at ϕ and cannot be tangential. This result follows from the fact that both $\bar{U}_p^{e_v}$ and $\bar{U}_A^{e'_v}$ will have the same slope along A's 45° line (at t and s respectively). Convexity of $\bar{U}_A^{e'_v}$ then implies that it will cut $\theta\bar{P}$ from below at ϕ and hence that Pareto preferred contracts are available at *lower* levels of effort. At lower levels of effort, the principal will bear some risk.

(3) The influence of increasingly "efficient" effort on the part of the agent is illustrated in Figure 6. The greater the efficiency of the agent's effort the less effort is required to achieve any particular value of q_1. In an Appendix we illustrate with a function $q_1 = 1 - e^{-\lambda\omega}$ where ω = effort and λ is an index of the efficiency of effort. As effort efficiency increases, the locus $\theta\bar{P}$ will shift outwards until it looks like the curve $\bar{\bar{\theta}}\bar{\bar{P}}$ in Figure 6. A Pareto efficient contract can then approach the "first best" solution along A's 45° line (say at δ).

(4) Where the agent is risk neutral and the principal is risk averse, the efficient contract will involve the agent shouldering the entire risk and the principal receiving a fee. This proposition is illustrated in Figure 7. The principal's indifference curves will now be convex and the agent's curves will be straight lines. The locus $\theta\bar{P}$ is constructed in precisely the same manner as before, however, and because A's indifference curves are no longer convex

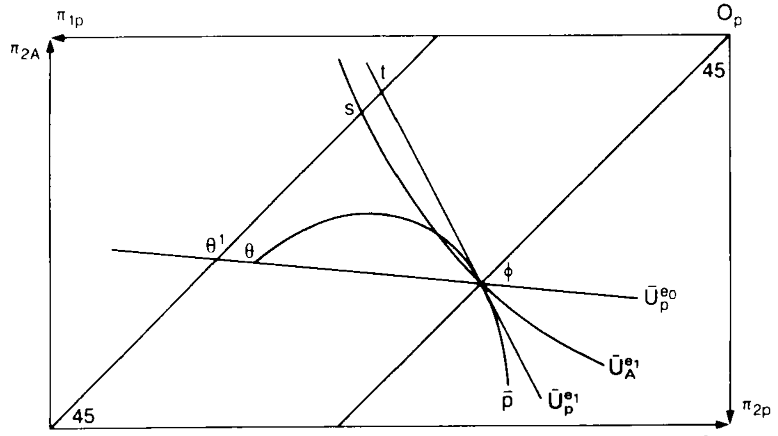

Figure 5. When A is risk averse and P is risk neutral, a contract at ϕ cannot be Pareto efficient. The relevant 'A' indifference curve will always cut $\theta\bar{P}$ from below at ϕ. This follows from the fact that both A's curve and P's curve through ϕ have the same slope along A's 45° line (at s and t).

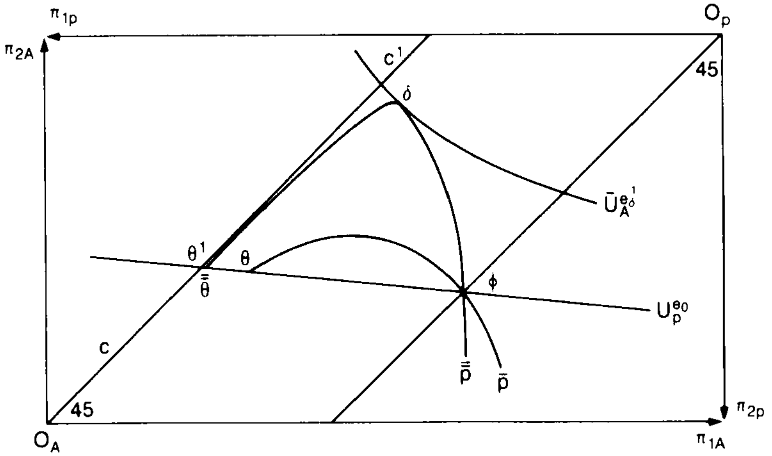

Figure 6. Increasing effort efficiency shifts the locus $\theta\bar{P}$ to $\bar{\bar{\theta}}\bar{\bar{P}}$ and the Pareto efficient contract approaches a first best along CC'.

an efficient contract will exist at ϕ. No point between the two 45° lines can be efficient under these circumstances. Suppose for example, that we imagined a point of tangency between $U_A^{e'_m}$ and $\theta\bar{P}$ at π on $q_1^{(m)}$. Such a position results in a contradiction, for it implies that there exists a P indifference curve $\bar{U}_p^{e_m}$ through ϕ applying to effort level m (and hence probability of outcome 1 $q_1^{(m)}$) which cuts $q_1^{(m)}$ at π. Since $U_A^{e'_m}$ and $\bar{U}_p^{e_m}$ have the same slope along P's 45° line and since P's indifference curves are now convex this is clearly impossible.

Efficient contracts where the agent is risk neutral and the principal is risk

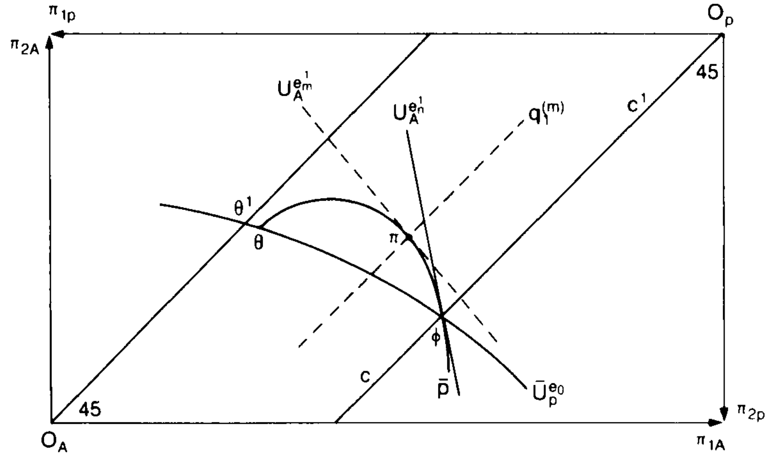

Figure 7. When the agent A is risk neutral and P is risk averse the Pareto Efficient contract is at ϕ. An efficient contract at π could not exist. It would imply the existence of a P indifference curve through both ϕ and π. Since the slopes of both A's and P's curves are the same along CC', and P is risk averse, no such P indifference curve could exist.

averse will therefore lie along P's 45° line and this will also result in efficient risk sharing. Pareto efficient contracts will be first best.

VI

Mutual Risk-Aversion

Thus far we have considered only cases in which one of the parties (either principal or agent) is risk neutral. We now consider what modifications are required to take account of those cases in which both principal and agent are risk-averse. Any tractable diagrammatic analysis will inevitably lack something of the complete generality characteristic of formal mathematical reasoning, but quite minor diagrammatic modifications are sufficient to accommodate a wide class of cases. In particular, we shall continue to assume that the preferences of both principal and agent are characterised by constant absolute risk aversion as defined by Arrow and Pratt. It is then a fairly simple matter to verify that the locus of efficient risk-sharing contracts will be defined by the following relationships (see Appendix 2)

$$\pi_{1A} - \pi_{2A} = (R_P/R_A)(\pi_{1P} - \pi_{2P}), \tag{6.1}$$

where R_P and R_A are the levels of Arrow–Pratt risk aversion experienced by P and A, respectively.

Diagrammatically this simply implies that the locus CC^1 will be a straight line which maintains a constant distance from A's 45° line and a constant distance from P's. Where $R_A = R_P$, CC^1 will simply bisect the distance between the 45° lines. Where $R_A > R_P$ the locus CC^1 will be closer to A's 45° line than P's, and vice-versa when $R_P > R_A$. In Figure 8, for example, the

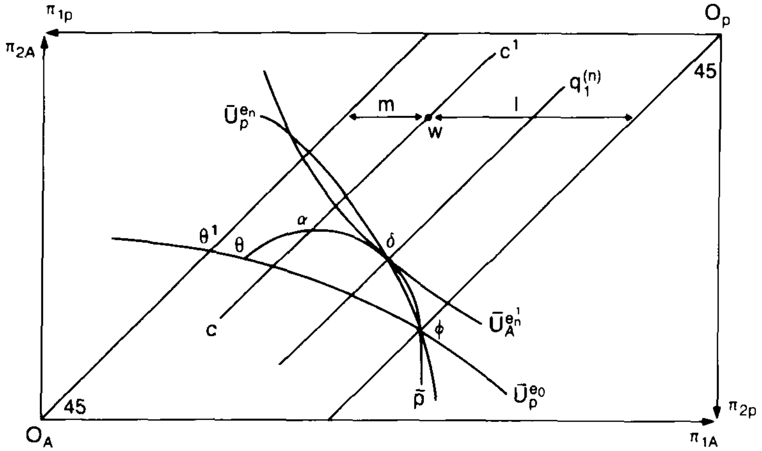

Figure 8. Both A and P are risk averse. If an appropriate A curve $U_a^{e'n}$ is tangent to $\theta\bar{p}$ (say at δ) it must *intersect* the relevant p curve. This implies that the efficient risk sharing locus CC' us to the left of δ. A contract at α would share risk efficiently but it would not be a Pareto efficient contract.

distance labelled $1 = \pi_{1P} - \pi_{2P}$ for the contract w. Similarly, $m = \pi_{1A} - \pi_{2A}$ for the same contract w . As R_P increases relative to R_A, expression 6.1 tells us that efficiency requires m to increase relative to 1. We move nearer to P's 45° line.

Interpretation of the rest of Figure 8 then follows closely that of Figure 4. A Pareto efficient contract at δ will involve the agent in greater effort than would be forthcoming in a contract along CC'. If an indifference curve $U_A^{e_n'}$ of the agent is to be tangential to $\theta\bar{P}$ it must *intersect* a principal's indifference curve $U_P^{e_n}$ from below. This implies that the locus of efficient risk sharing arrangements lies to the left of δ and that the Pareto efficient contract must involve the sacrifice of some of these benefits in the interests of inducing greater effort. A contract at α, for example, on $\theta\bar{P}$ would share risk efficiently but it would not be a Pareto efficient contract.

VII

Opposing Risk Preferences

The possibility of risk-preferring individuals suggests that the diagrammatic apparatus developed here might be used to indicate how conclusions would be altered if one of the parties were risk preferring. In Figure 9, the agent is slightly risk preferring and the principal is sufficiently risk averse to ensure a locus of "first best" tangency points. Clearly if P were risk neutral or only slightly risk averse, the efficient sharing of risk would involve only corner solutions.

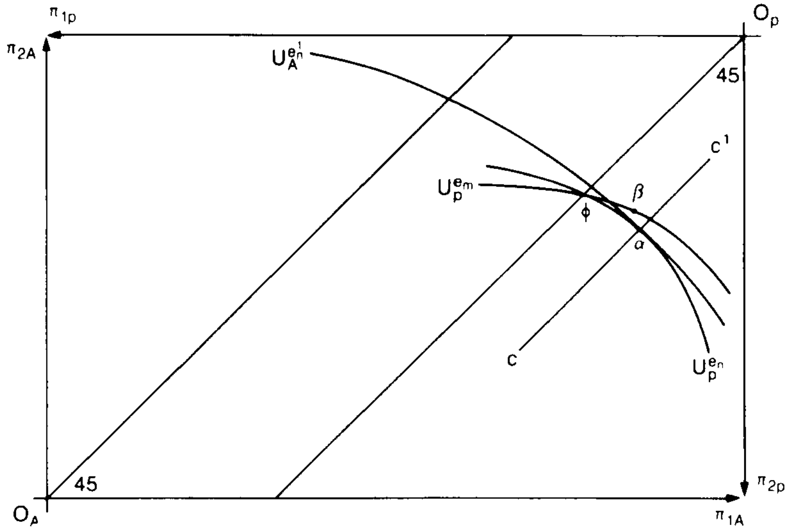

Figure 9. A is slightly risk preferring, P is risk averse. The efficient risk-sharing locus is CC'. The interests of A and P conflict concerning the most attractive outcome. *Lower* effort on the part of the agent and hence a contract at β will permit Pareto improvements on the contract at α. A contract at α will be attainable only if the ability of the agent to increase the probability of outcome 1 is sufficiently low and declines to zero before CC'.

As drawn, the locus CC' once more indicates "first best" solutions to the risk sharing problem. Notice that CC' is no longer between the two 45° lines but is now to the right of P's. Along CC' therefore, it is evident that the interests of principal and agent conflict significantly. For A's interests lie in increasing the probability that outcome 1 occurs, whereas given a contract on CC', P's interests now lie in securing an increase in the probability of outcome 2. Not surprisingly this basic conflict of interest will restrict the ability of A and P to secure risk sharing benefits. Consider a contract at α on CC' at a point of tangency between $U_A^{e'_n}$ and $U_p^{e_n}$. If the agent's effort is effective in increasing the probability of outcome 1 some Pareto improvement is available from reducing this incentive and moving away from CC'. A level of effort $e'_m < e'_n$ on the part of the agent would change the principal's indifference curve to $U_p^{e_m}$. A contract (say) at β on $U_P^{e_m}$ would be Pareto preferred to α. We conclude that a contract on CC' will only be possible if the marginal influence of the agent's effort on the probability of outcome 1 declines sufficiently quickly and reaches zero at or before CC'. If, for example, we consider a contract at ϕ on P's 45° line and suppose the probability of outcome 1 to be entirely uninfluenced by further increases in $\pi_{1A}-\pi_{2A}$ a contract at α would clearly be an attainable first best position. If, on the other hand, the agent's effort is always effective, *some* departure from ϕ will be optimal as risk sharing benefits are bought at the cost of some "perverse" incentives, but the first best locus will not be attained.

Finally Figure 10 reverses the roles of the parties and illustrates the case of a risk preferring principal and risk-averse agent. Once more the first best contract locus implies the existence of incentives for the agent systematically to frustrate the objectives of the principal. If sabotage were easily observable or this kind of effort entirely inefficient, a risk sharing optimum might be attained at α. This, however, would be to sacrifice the effort-providing

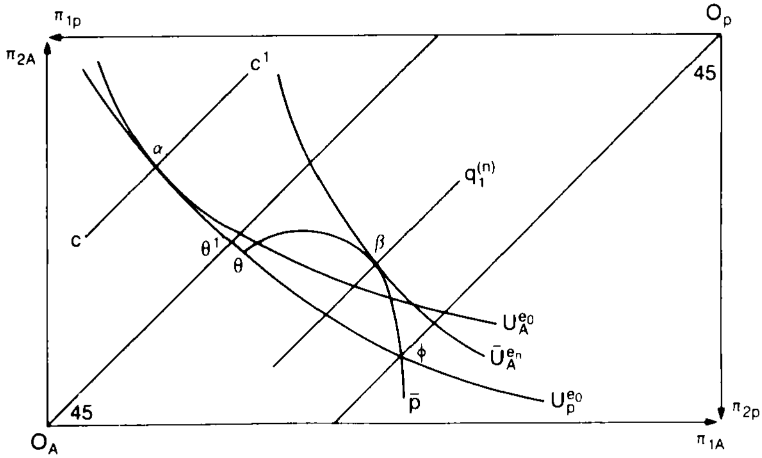

Figure 10. A is risk averse. P is risk preferring. Contracts along CC' imply an incentive for the agent to frustrate the principal. If sabotage is perfectly observable, a contract at α will achieve risk-sharing benefits but no effort. The contract at β is Pareto preferred if $\bar{U}_A > U_A$.

services of the agent altogether. To obtain these, a contract along $\theta\bar{P}$ would be necessary. At β, for example, Pareto efficient effort is being supplied, and providing $\bar{U}_A > U_A$, a contract at β will be Pareto preferred to a contract at α even when sabotage is ruled out. It is worth noting that in this case, perfectly efficient effort on the part of the agent will not result in a contract near the "first-best" position. Even highly efficient effort will only permit contracts close to A's 45° line. Shavell's (1979) conclusion concerning the efficiency of effort would appear to apply therefore to the case of risk-averse principal and agent, but not to the case of a risk-preferring principal.

VIII

APPLICATIONS OF PRINCIPAL-AGENT THEORY

The primary objective of this paper has been to consider the principal-agent problem using a familiar diagrammatic apparatus, and to reinforce the intuitive appeal of some of the main results. In this final section we refer briefly to several areas of applied analysis in which principal-agent theory has been used in recent years.

1. *Labour contracts*

Let the principal be a risk-neutral employer and the agent a risk-averse employee. Note that to the lawyer this statement would not make sense, since an employee is not an agent. The terms principal and agent are used in this paper in a much looser way than would be permitted by the lawyer. Interpreted in this light, point θ' in Figure 4 would represent a contract based upon a "time rate" for the employee, a payment independent of the actual outcome. Such a contract would share risk efficiently but would leave severe problems of incentives unless effort or state of the world were observable and the wage were made conditional upon these. The "implicit contracts" literature revolves around this possibility and postulates that the employer provides insurance to the employee thereby insulating the latter from excessive variation in income levels and producing the phenomenon of "wage stickiness" over the business cycle. Azariadis and Stiglitz (1983) provide an appraisal of the modern literature but the idea can be found in Knight's (1921) investigation of the role of the entrepreneur. "The confident and venturesome assume the risk or insure the doubtful and timid by guaranteeing to the latter a specified income in return for an assignment of the actual result." (pp. 269–270).

In contrast point Φ in figure 4 would represent an arrangement by which the employee keeps what is produced but pays a fee to the employer for the right to work in the firm. It is equivalent to a type of quota piece-rate, with output below the quota representing the fee to the employer. We noted earlier that risk-averse employees would not accept this type of contract. Some sharing in the risk of the enterprise is predictable where monitoring costs are high however. Points on a diagonal line between O_A and O_P for example, would designate contracts which specified that a certain fraction of the outcome should accrue to each party. Stiglitz (1975) investigates the class of linear incentive structures where the outcome alone is observable. A linear incentive structure consists of a time rate, independent of the actual outcome, plus a piece rate ($\Pi_A = w + r\Pi$ where Π_A is A's remuneration, w is the time rate, Π is the outcome and $0 < r < 1$). He shows that the efficient piece rate is higher, the lower the degree of risk aversion (p. 560). Indeed in the case of risk-neutral workers, point Φ in Figure 4 is efficient as we have seen, the time rate becomes negative (the 'fee' mentioned at the beginning of this paragraph) and $r = 1$.

2. *Contracts between firms*

The analysis of principal and agent has also been used in modern discussions of industrial integration. If firm A is the supplier and firm P the purchaser of some component, a contract at θ' would represent a "cost plus" agreement by which A is assured of a certain profit and firm P takes the risk. The incentive problems of these arrangements will lead to attempts at close monitoring and possibly, where a relationship is likely to be a continuing one, to full integration. If monitoring is very costly we will observe contracts closer to point Φ. Point Φ might be interpreted as a "fixed-price" contract with the supplying firm now shouldering the risk. Under risk neutrality this solution would once more be efficient. Franchise arrangements by which firm A pays a fee to firm P to undertake a certain activity also correspond with point Φ. Rubin (1978) criticises traditional explanations of franchising which emphasise capital market imperfections and the tapping of new funds provided by franchisees, and prefers an approach based upon the costs of monitoring distant operations. Franchising provides managerial incentives, although risk aversion on the part of franchisees will count against. Rubin notes that in areas where monitoring costs are lower (e.g. a fast food chain in a densely populated urban area) professional managers may replace the franchisee.

3. *The division of ownership from control*

Here person A is the manager and P the shareholder or other "owner". The conventional argument based upon the work of Berle and Means (1932) is that a contract will be fixed at θ' and that this will leave the managers with little incentive to run the business efficiently. More recent work by Masson (1971), Lewellen (1969), and Lewellen and Huntsman (1970) suggests however, that the manager's reward is not independent of the performance of the firm and writers such as Demsetz (1983) have drawn attention to the extent of managerial shareholdings. Point Φ in Figure 4 might be considered as approximating the world of the "management buy out" with person P receiving a predetermined payment on bond finance. In fact, of course, this would be an oversimplified view since very highly leveraged financial structures create moral hazard problems of their own as discussed by Jensen and Meckling (1976). Control problems are also important in the nationalised industry sector. Here the problem is to think of ways of moving away from θ' in a world without exchangeable residual claims. A contract somewhere on $\theta\bar{P}$ would require managerial remuneration to be linked to some observable measure of performance.

4. *The non-profit enterprise*

Reinterpreting Figure 4 again, we might regard A as a supplier and P as a consumer. In this case a contract at θ' could be seen as applying to a non-profit enterprise. The consumer takes the risk and the supplier's costs and remuneration must be "reasonable". Point Φ represents the usual for-profit case where the supplier is free to choose any methods of production he wishes and make as much profit as he can. Easley and O'Hara (1983) note that if capital markets provide risk protection to managers so that risk neutrality prevails a for-profit contract will produce the first best outcome (point Φ once more). They admit that this suggests a somewhat limited role for the non-profit institution (a device for coping with managerial risk-aversion) but then follow Hansmann's (1980) argument that in circumstances where consumer benefits are themselves not observable a non-profit contract may be advantageous. Examples include charitable donations to famine relief, or the purchase of nursing services on behalf of someone else who is incapable of reporting on their quality.

These brief comments are merely designed to illustrate some of the ways in which economic theorists have applied the principal-agent approach, and an exhaustive survey has not been attempted. Enough has been said, however, to indicate the ways in which the theoretical literature on contracts is beginning to influence our ideas on the determinants of industrial structure, institutional design, the employment relation, and many other areas of inquiry.

APPENDIX 1

Let the utility function of the agent be represented by

$$U_A(\pi_A - \omega) = a - be^{-R_A(\pi_A - \omega)}$$

where R_A is the Arrow–Pratt index of absolute risk-aversion and ω represents resources devoted to changing the probability of outcome 1. A's expected utility can be written

$$E[U_A] = q_1(\omega)U_A(\pi_{1A} - \omega) + (1 - q_1(\omega))U_A(\pi_{2A} - \omega).$$

Differentiating with respect to ω and setting the result equal to zero we obtain as a first order condition (for interior solutions with $\omega > 0$)

$$[1 - e^{-R_A(\pi_{1A} - \pi_{2A})}][R_A q_1(\omega) + q_1'(\omega)] = R_A. \qquad (1)$$

Let $q_1(\omega) = 1 - e^{-\lambda\omega}$ where λ is an index of the efficiency of effort. Substituting for $q_1(\omega)$ and $q_1'(\omega)$ in (1) we obtain

$$e^{R_A(\pi_{1A}-\pi_{2A})} = 1 + \left(\frac{R_A}{\lambda - R_A}\right)e^{\lambda\omega}. \tag{2}$$

If $\lambda < R_A$, $\omega = 0$ for all values of $\pi_{1A} - \pi_{2A}$. Hence we assume $\lambda > R_A$. From (2) it is seen that for *given* R_A and λ, higher levels of ω require higher values of $\pi_{1A} - \pi_{2A}$. Thus

$$\frac{\partial\pi^*}{\partial\omega} = \frac{\lambda}{\lambda - R_A}e^{\lambda\omega - R_A\pi^*} > 0 \tag{3}$$

where $\pi^* = \pi_{1A} - \pi_{2A}$. Further

$$\frac{\partial^2\pi^*}{\partial\omega^2} = \frac{\partial\pi^*}{\partial\omega}\left(\lambda - R_A\frac{\partial\pi^*}{\partial\omega}\right). \tag{4}$$

From (2) and (3) it can be seen that

$$\frac{\partial\pi^*}{\partial\omega} = \frac{\lambda}{R_A}(1 - e^{-R_A\pi^*}).$$

Hence $\partial\pi^*/\partial\omega < \lambda/R_A$ and (from (4)) $\partial^2\pi^*/\partial\omega^2 > 0$.

For a given probability q_1^*, $\partial\omega/\partial\lambda = -(\omega/\lambda) < 0$. As the efficiency of effort increases, effort necessary to achieve q_1^* declines. Lower effort then requires a lower value of $\pi_{1A} - \pi_{2A}$. Thus

$$\frac{\partial\pi^*}{\partial\lambda_{(q_1\ \text{constant})}} < 0. \tag{5}$$

To find an efficient contract between agent and principal we maximise $E[U_A]$ subject to the constraints $E[U_p] = \bar{E}$ and expression (2) which we write in implicit form $A(\omega, \pi^*)$. $U_p = U_p(\pi_p)$ where $\pi_p = \pi - \pi_A$ and π represents total available resources.

$$\text{Max}\, L(\pi_{1A}, \pi_{2A}, \omega, m_1, m_2) = E[U_A] - m_1(E[U_P] - \bar{E}) - m_2A(\omega, \pi^*). \tag{6}$$

Diagrammatically the constraint $E[U_p] = \bar{E}$ is represented by the series of indifference curves radiating out of ϕ in Figure 3, while the constraint $A(\omega, \pi^*)$ is represented by the series of parallel lines $q_1^{(\omega)}$ in the same diagram. Thus the locus $\theta\bar{P}$ gives us points in the diagram which satisfy the two constraints for the appropriate levels of effort. Maximisation of $E[U_A]$ subject to the two constraints combined is then illustrated in Figure 4.

The problem illustrated in these diagrams is therefore a tractable version of Holmström's (1979, p. 77) approach. The main differences are:

(a) For diagrammatic clarity it is easier to maximise the utility of the agent rather than (in Holmström's case) the principal.

(b) We restrict the diagram to the class of constant absolute risk aversion utility functions.

(c) Since there are only two outcomes and $q_1'(\omega) > 0$, effort on the part of the agent clearly satisfies Holmström's requirement that "it will shift the distribution of (the outcome) to the right in the sense of first-order stochastic dominance" p. 77.

(d) Holmström assumes that the agent's utility function is additively separable in wealth and effort. Our assumption is no less restrictive but does not imply additive separability.

The particular form of utility function assumed combined with the function $q_1(\omega)$ permits us to draw the constant effort lines with slopes of unity. Such assumptions are not absolutely essential for the diagrammatic analysis however. For example, a utility function of the form (say)

$$U_A = a - b\mathrm{e}^{-R_A\pi_A} - c\mathrm{e}^{\omega}$$

might equally well have been used. Combined with the same function $q_1(\omega)$ used earlier $E[U_A]$ is concave in ω, and the first order condition for utility maximization becomes

$$\mathrm{e}^{\omega(1+\lambda)} = \frac{\lambda b}{c}(\mathrm{e}^{-R_A\pi_{2A}} - \mathrm{e}^{-R_A\pi_{1A}}).$$

The locus of constant effort will have a slope

$$\frac{\partial\pi_{2A}}{\partial\pi_{1A}} = \mathrm{e}^{-R_A\pi^*} < 1 \quad (\text{for } R_A, \pi^* > 0).$$

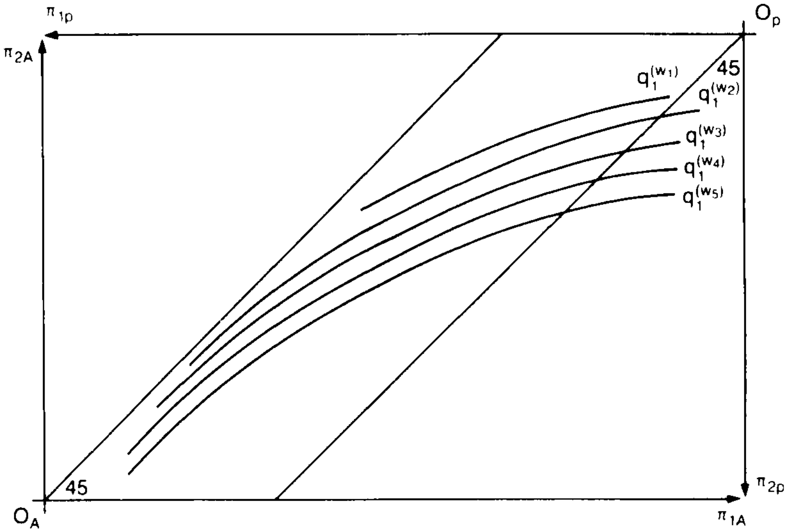

Figure A1. Using a utility function such as $U_A = a - be^{-R_A\pi_A} - ce^{\omega}$ the constant effort curves would no longer be straight lines with a slope of unity but would behave as shown in the figure. For a given value of $\pi^* = \pi_{1A} - \pi_{2A}$ the slope of every curve is the same.

Further

$$\frac{\partial^2 \pi_{2A}}{\partial \pi_{1A}^2} = R_A e^{-R_A \pi^*}\left(\frac{\partial \pi_{2A}}{\partial \pi_{1A}} - 1\right) < 0.$$

Figure A1 illustrates the shape of the curves $q_1^{(\omega)}$ under these conditions.

In the case of a risk-preferring agent (see Section 7) we use the function

$$U_A = a + be^{-R_A \pi_A} - ce^{\omega}, \quad (R_A < 0).$$

$E[U_A]$ is still concave in ω, but we now have

$$\frac{\partial \pi_{2A}}{\partial_{1A}} > 1, \qquad \frac{\partial^2 \pi_{2A}}{\partial \pi_{1A}^2} < 0.$$

It is this formulation which underlies the analysis of the first part of Section 7.

APPENDIX 2

Both principal and agent have utility functions of the form

$$U(\pi) = a - be^{-R\pi}.$$

If π_1 and π_2 are money outcomes and q_1 = probability of outcome 1, then

$$E[U] = q_1 U(\pi_1) + (1 - q_1)U(\pi_2).$$

The slope of a constant expected utility indifference curve is

$$\frac{\partial \pi_2}{\partial \pi_1} = -\frac{q_1}{1 - q_1} e^{-R(\pi_1 - \pi_2)}.$$

Using subscripts A and P to denote persons as in the text we find that points of tangency between A's curves and P's curves occur where

$$e^{-R_A(\pi_{1A} - \pi_{2A})} = e^{-R_P(\pi_{1P} - \pi_{2P})}$$

i.e. where

$$\pi_{1A} - \pi_{2A} = \frac{R_P}{R_A}(\pi_{1P} - \pi_{2P}).$$

APPENDIX 3

Computer simulations of the locus $\theta \bar{P}$

Computation by Dr Anne Walton, University of Buckingham.

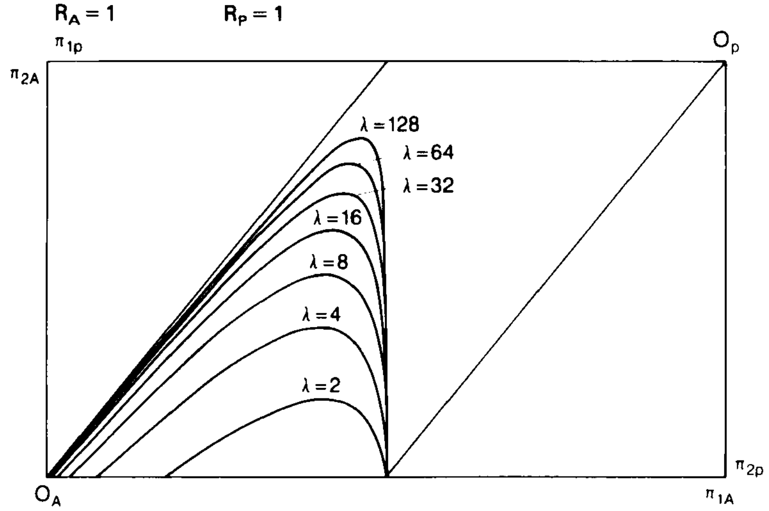

Figure A2. (A) The effect of effort efficiency (λ).

All simulations are based upon the following assumptions:

1. $U_A(\Pi_A - w) = a - e^{-R_A(\Pi_A - w)}$
2. $U_p(\Pi_p) = a - e^{-R_p \Pi_p}$
3. $q_1(w) = 1 - e^{-\lambda w}$
4. $E(U_p) = \Pi_2$
5. $\Pi_1 - \Pi_2 = 2$

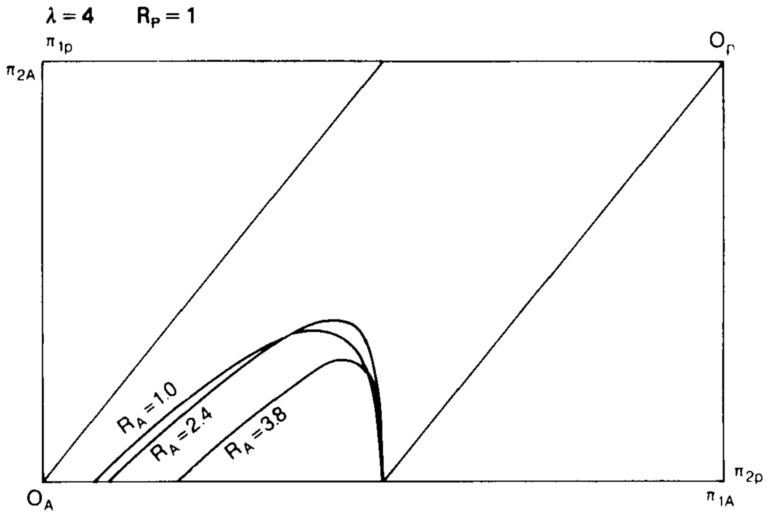

Figure A3. (B) The effect of the agent's risk aversion (R_A).

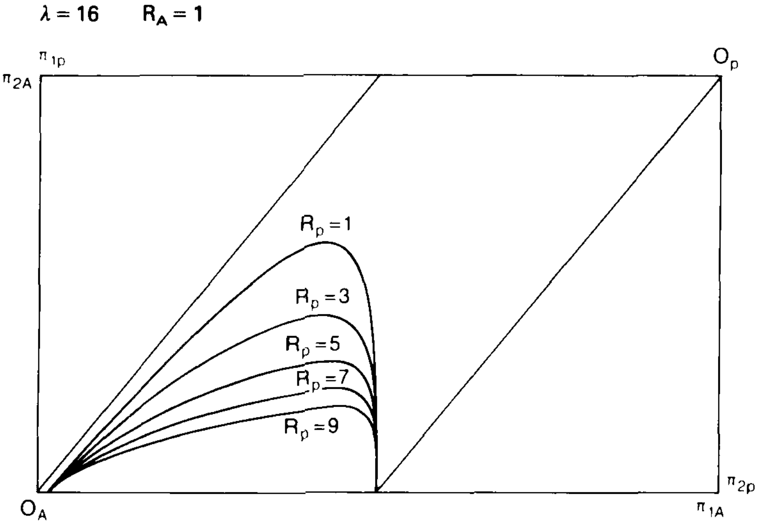

Figure A4. (C) The effect of the principal's risk aversion (R_p).

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